# Studying a Nonlinear System Using Linear Methods

Given a NL system:

$\dot{x} = y$

$\dot{y} = (1 - x^2) + x$

The EP is (0, 0)

The Jacobian is:

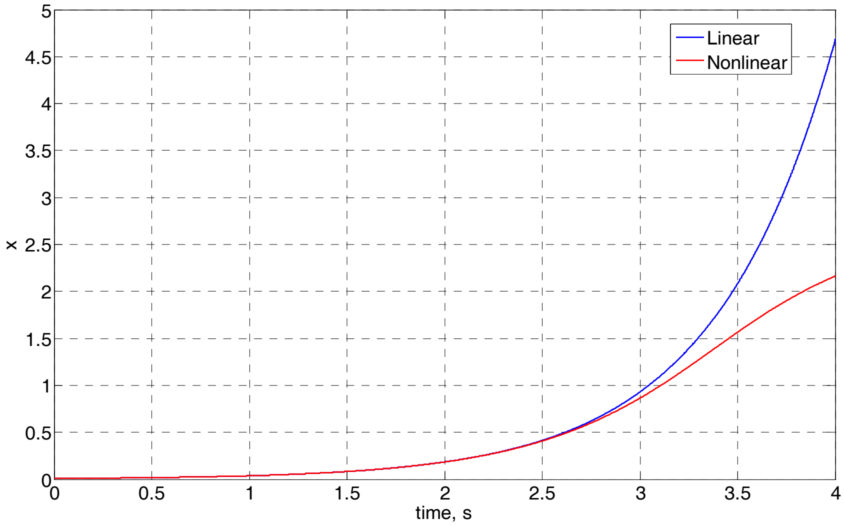
$$J = \begin{bmatrix} 0 & 1 \\ -2xy + 1 & 1 - x^2 \end{bmatrix}$$

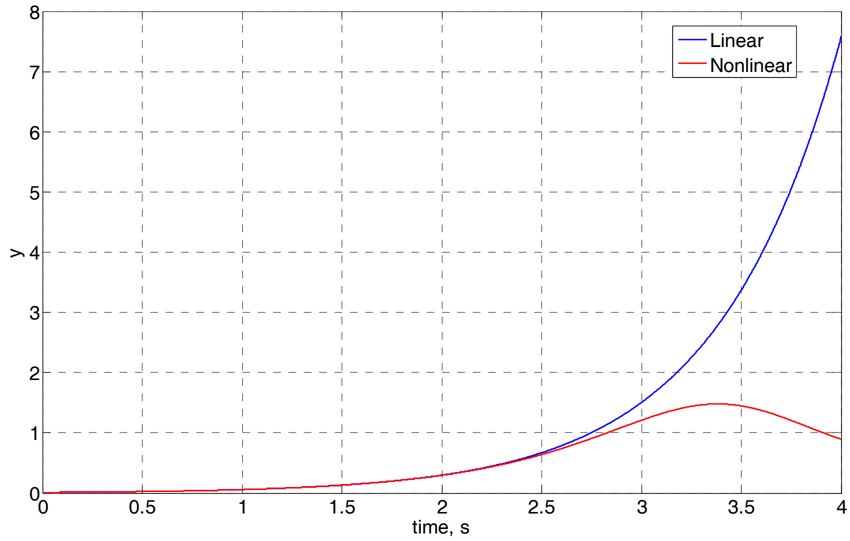
Hence the Jacobian at the EP is:

$J = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ with eigenvalues: $\lambda_1 = \text{-}0.618$, $\lambda_2 = 1.618$

And eigenvectors $e_1 = \begin{bmatrix} \text{-}0.85065080835204 \\ 0.525731112119134 \end{bmatrix}$ , $e_2 = \begin{bmatrix} 0.525731112119134 \\ 0.85065080835204 \end{bmatrix}$

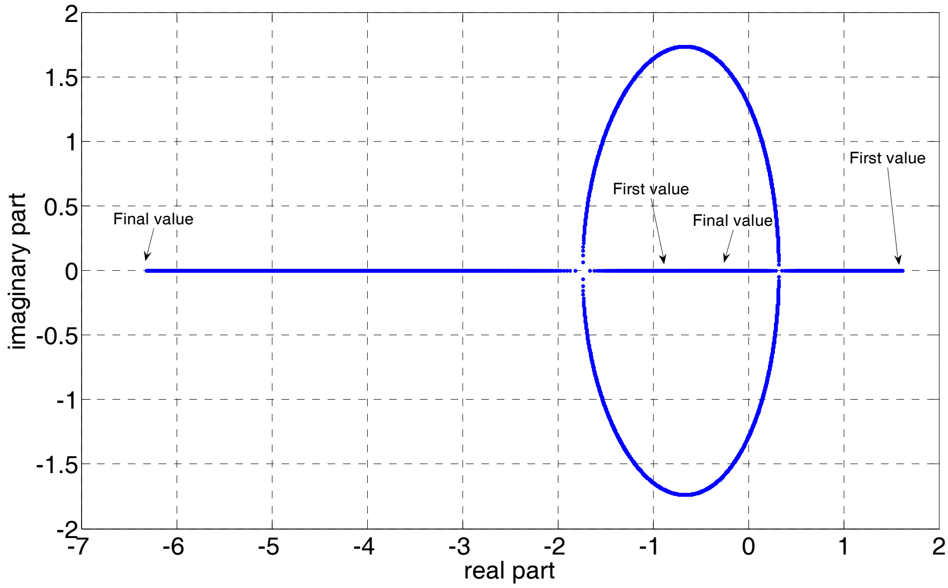
The response of the linearised and nonlinear system starting from $[0.01 \quad 0.01]$ is:



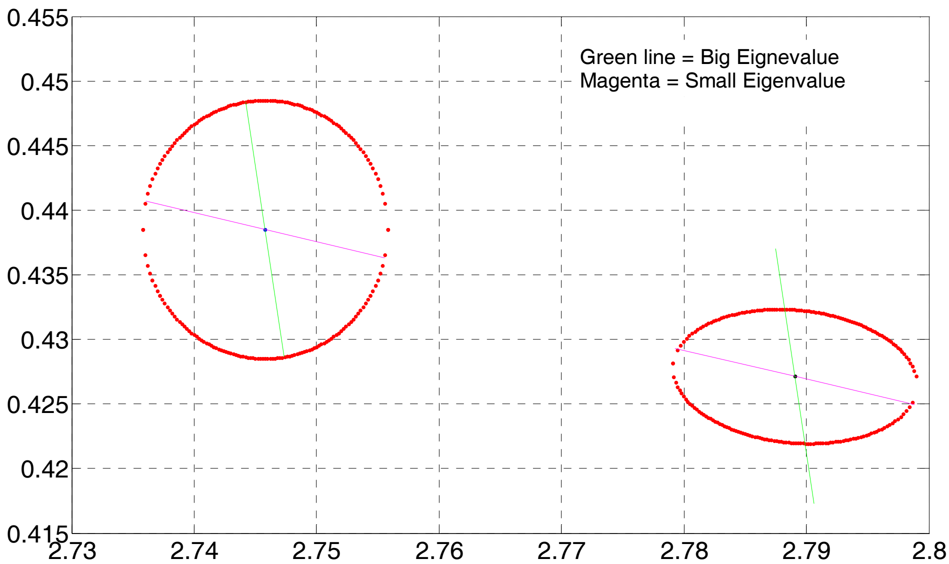




Hence we can see that for a small interval the 2 systems coincide!

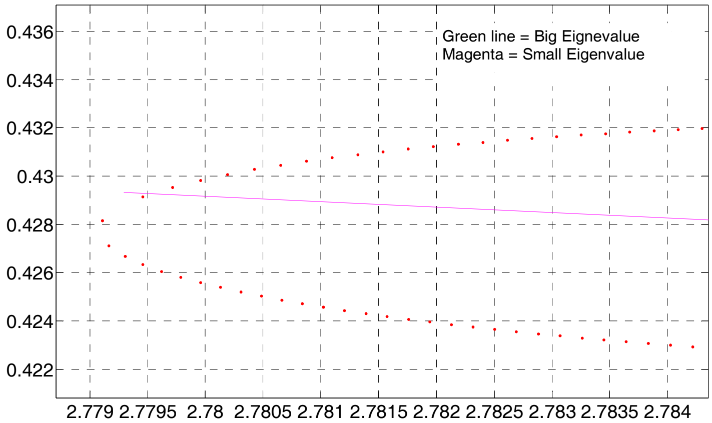
Now, using a simple Simulink model I calculate the values of the Jacobian and the eigenvalues at various points of the **nonlinear** response:



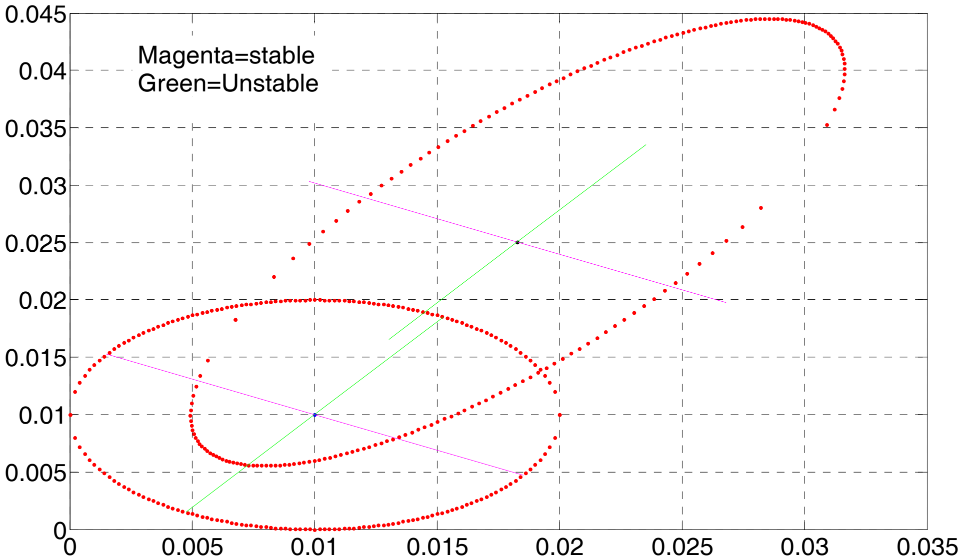

So what has happened is we started from a saddle point (as expected near the EP), then the 2 eigenvalues approached, the negative became also positive and at some point they became unstable and complex. Then gradually they became stable complex and then both of them stable. Of course this does not mean that the system is stable at this “final” point. It just means that if we take a small circle of ICs near this final point then after a small interval this circle will shrink and will be an ellipse with the major axis being in the direction of the eigenvector with the smallest eigenvalue.
So let’s test that:
The final point is $[2.74578350743219 \quad 0.438510769417 49]^T$ and let’s take a small radius of 0.1. The result is:




This indeed validates the previous statement. At this point it has to be mentioned that there is also a reduction along the magenta line but the eigenvalue here is -0.22 so a very small change is expected which can be seen if the graph is zoomed in:

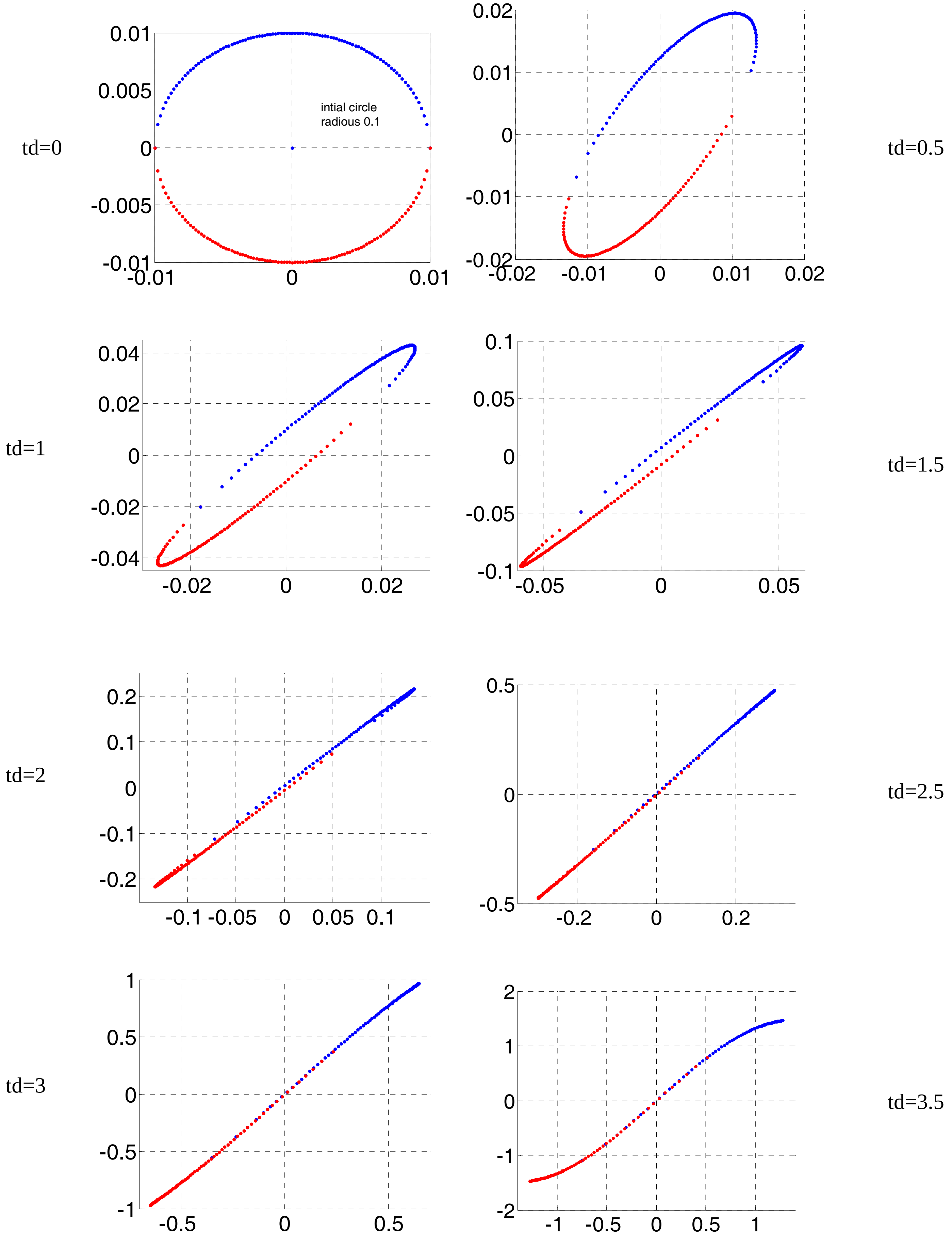



The initial point is a saddle, so I expect an expansion in one direction and a contraction in the other.




Note: The original circle looks like an ellipse but is not. The magenta and green lines are the eigenvectors at the initial point. I just plotted them again at the final to see the reduction.

Now, lets start from a circle around the saddle EP and lets see how this circle will evolve over time: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intial circle
radious 0.1
td=0.5
td=1
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td=3
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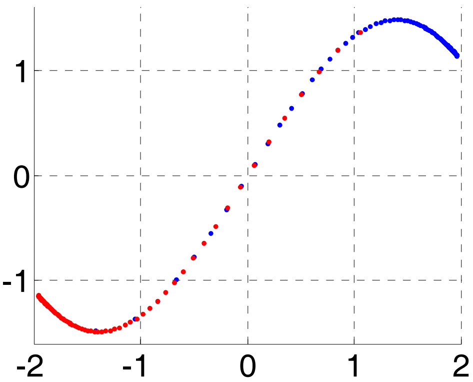
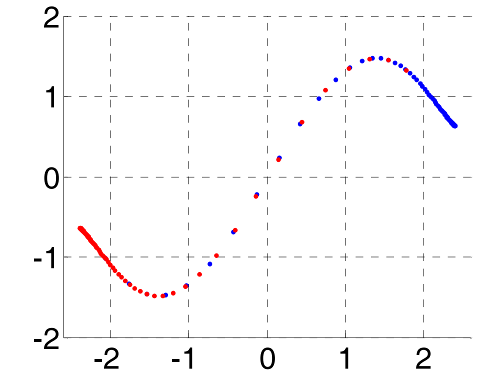
td=4

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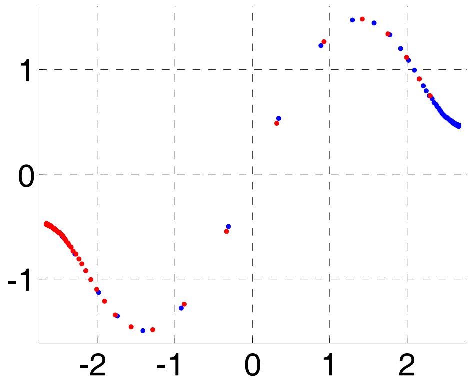


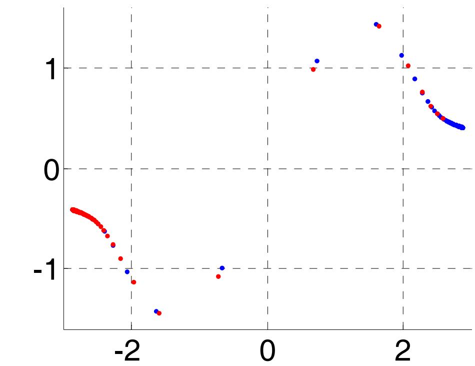


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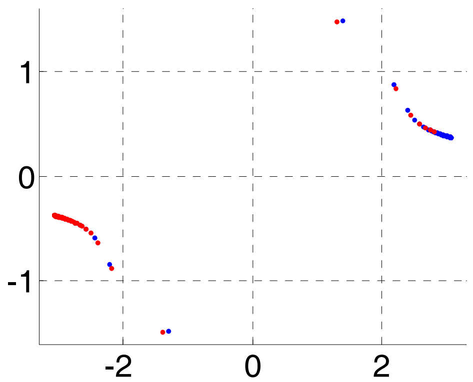


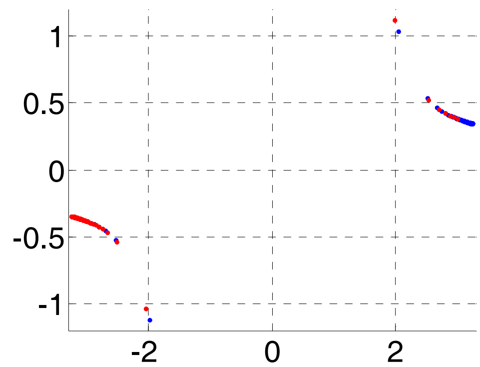


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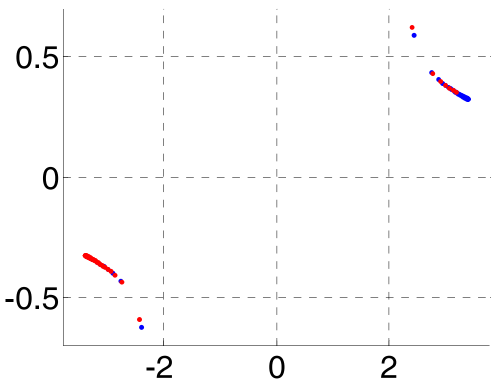


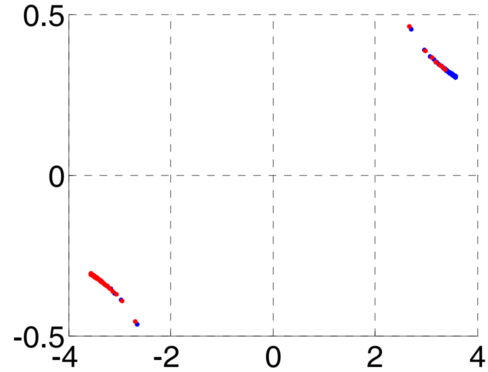


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